Additional Comments on Spatially Growing Disturbances in Liquid Films

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In a recent note Agrawal and Lin (1975) solve the spatial formulation of the linear stability problem for falling film flow via a regular perturbation expansion in the dimensionless angular frequency ω. They compare the spatial amplification factor predicted by this solution with that predicted by the solution of Benjamin (1957) and Yih (1963) to the temporal formulation of this linear stability problem. In order to make this comparison they transform the temporal amplification factor predicted by Benjamin and Yih to an equivalent spatial amplification factor via Gaster's (1965) transformation. This transformed temporal amplification factor is found to be identical to the spatial amplification factor of Agrawal and Lin. On the basis of these results Agrawal and Lin state "This contradicts Krantz's contention . . . . . that the solution for the spatial case cannot be obtained from that for the temporal case, since, according to Krantz, the falling film is highly unstable." In this note we wish to point out that the proof presented by Agrawal and Lin contains nothing new regarding the equivalence of the spatial and temporal formulations of this linear stability problem and certainly does not discredit any of the results presented in Krantz and Owens (1973) nor the later comments of Krantz (1975).

Note that the solution of Agrawal and Lin involves a regular perturbation expansion in the parameter ω; hence this solution only applies for $\omega << 1$. Table 1 shows the values of the wave number α_r, phase velocity c_r, angular frequency ω, and spatial amplification factor $-\alpha_i$ for the data presented in Figures 4 and 5 in Krantz and Owens (1973). Note that in no case is $\omega << 1$; thus the long wave solution of Agrawal and Lin cannot be used to conclude anything concerning the equivalence of the spatial and temporal formulations for the data presented by Krantz and Owens. These data apply to relatively short waves which can be both highly amplified and dispersive.

Indeed, the results obtained by Agrawal and Lin are not surprising. Since long waves are weakly amplified, the theorem of Gaster (1965) indicates that the spatial and temporal formulations will be equivalent. Furthermore since long waves are nondispersive [$c_r = 3 + O(\alpha_r^2)$], the transformation of Schubauer and Skramstad (1949) also is applicable. This latter transformation, which is a special case of the more general transformation of Gaster applicable only to nondispersive waves, in effect is used by Agrawal and Lin since $\partial\omega/\partial\alpha_r = c_r = 3$ for their long wave solution. Unfortunately the perturbation scheme used by Agrawal and Lin does not converge rapidly and may not converge at all for the range of highly amplified frequencies encountered in the film flow of the White Oils used by Krantz and Goren (1971). A more generally applicable exact solution to this linear stability problem is clearly desirable. Recently Shuler (1974) has developed an exact asymptotic solution to the spatial formulation of this linear stability problem via a power series expansion in the independent variable y. This latter solution retains terms of seventh-order in α and applies to both two- and three-dimensional disturbances. When this exact solution is truncated to retain only terms of second-order in α[assuming $N_{We} = O(1/\alpha_r^2)$ as do Agrawal and Lin], it is found to include the solution of Agrawal and Lin as a special case [Equation (V-43) on page 111 in Shuler (1974) is identical to Equation (5) in Agrawal and Lin (1975)]. Shuler found that for long waves the two formulations were equivalent. However, for the shorter waves characteristic of the data of Krantz and Goren, this exact solution indicates that the two formulations are not equivalent. Thus the contention of Krantz and Owens concerning the equivalence of the spatial and temporal formulations of this linear stability problem is in no way contradicted by the recent note of Agrawal and Lin.

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TABLE 1. DATA FOR FIGURES 4 AND 5 IN KRANTZ AND OWENS (1973)

$N_{Re} = 0.708$, $N_\zeta = 10.2$, $N_{We}^{-1} = 0.0546$, $\beta = 90°$

α_r	c_r	ω	$-\alpha_i$
0.143	3.04	0.435	0.00401
0.143	3.04	0.435	0.00493
0.162	3.06	0.496	0.00632
0.188	2.98	0.560	0.00755
0.206	3.00	0.618	0.00860
0.226	3.02	0.683	0.00957
0.256	2.91	0.745	0.00990
0.276	2.93	0.809	0.00945
0.304	2.87	0.872	0.00833
0.325	2.87	0.933	0.00638
0.358	2.78	0.995	0.00340
0.377	2.80	1.06	−0.00285
0.410	2.74	1.12	−0.00912
0.422	2.80	1.18	−0.0169

$N_{Re} = 1.29$, $N_\zeta = 2.89$, $N_{We}^{-1} = 0.527$, $\beta = 90°$

α_r	c_r	ω	$-\alpha_i$
0.293	2.89	0.847	0.0361
0.356	2.77	0.986	0.0401
0.380	2.78	1.06	0.0424
0.408	2.76	1.13	0.0478
0.458	2.61	1.20	0.0491
0.477	2.66	1.27	0.0469
0.523	2.56	1.34	0.0456
0.548	2.57	1.41	0.0433
0.597	2.48	1.48	0.0389
0.643	2.41	1.55	0.0347
0.726	2.33	1.69	0.0121

NOTATION

c_r = dimensionless phase velocity, $c_r^*/\overline{U}$
g = acceleration of gravity
h_0 = film thickness of basic flow
N_{Re} = Reynolds number, $\overline{U}h_0/\nu$
N_{We} = Weber number, $\sigma/\rho h_0 \overline{U}^2$
N_ζ = surface tension group, $(\sigma/\rho)\ (3/g\nu^4)^{1/3}$
$\overline{U}$ = average basic flow velocity
y = cross-stream coordinate

Greek Letters

α = dimensionless complex wave number, $2\pi h_0/\lambda$
α_i = spatial amplification factor
α_r = real wave number
β = angle of plane to horizontal
λ = complex wave length
ν = kinematic viscosity
ρ = density
σ = surface tension
ω = dimensionless angular frequency, $\omega^* h_0/\overline{U}$

Superscript

$*$ = dimensional quantity

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An Analysis of Turbulent Pipe Flow for a Viscoelastic Fluid

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A note by the author (1974) showed that the friction factor for power law non-Newtonian fluids for turbulent flow in a pipe corresponds to the velocity profile slope at $y^+ = 12$. This note extends this work to an analysis for a viscoelastic fluid.

FRICTION FACTOR RATIO

The prior note showed that the wall region frequency relationship for Newtonian fluids in turbulent flow

$$\frac{u^{*2}\,t}{\nu} = 338 \qquad (1)$$

is also applicable to power law non-Newtonian fluids. Elastic fluids appear to have the property of decreasing this frequency which results in drag reduction at the pipe wall. The momentum equation for the boundary layer can be written for Newtonian and elastic fluids as

$$\frac{\tau}{\rho} = 0.332\,U_{\infty N}^2 \sqrt{\frac{\nu}{U_{\infty N}x}} \qquad (2)$$

and

$$\frac{\tau}{\rho} = 0.332\,U_{\infty E}^2 \sqrt{\frac{\nu_1}{U_{\infty E}x}} \qquad (3)$$

Earlier work (1973) has shown that

$$x = 338\,\nu/u^* \qquad (4)$$

for Newtonian fluids corresponding to the average velocity in the laminar layer of $y^+ = 2.0$. A general form of Equation (4)

$$x = \alpha\nu_2/u^* \qquad (5)$$

can be used to represent frequency and laminar layer thicknesses that could occur with elastic fluids in turbulent flow. Combining Equations (2) and (4) yields

$$U^+_{\infty N} = 14.5 \qquad (6)$$

and Equations (3) and (5):

$$U^+_{\infty E} = 2.08\,\alpha_E^{1/3}\,(\nu_2/\nu_1)^{1/3} \qquad (7)$$

The friction factor relationship corresponding to the dimensionless shearing stress for a developing boundary layer on a flat plate provides for Newtonian fluids

$$f_N = \frac{0.036}{\sqrt{U^+_{\infty N}}} \qquad (8)$$

with substitution of Equation (4) and

$$f_E = \frac{0.664}{(\alpha_E U^+_{\infty E}\,\nu_2/\nu_1)^{1/2}} \qquad (9)$$

for elastic fluids with substitution of Equation (5). Combination of Equations (6), (7), (8), and (9) yields the friction factor ratio